Revision of the "Revised Advisory Note No. 3: Thermodynamic Derivatives from IAPWS Formulations"

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Motivation and Aim of the Advisory Note No. 3

Thermodynamic Derivatives as
\[ \left( \frac{\partial h}{\partial p} \right)_v, \left( \frac{\partial u}{\partial p} \right)_v, \left( \frac{\partial s}{\partial p} \right)_v, \left( \frac{\partial T}{\partial p} \right)_h, \left( \frac{\partial T}{\partial p} \right)_s, \left( \frac{\partial v}{\partial h} \right)_p, \left( \frac{\partial v}{\partial s} \right)_p \ldots \]

are used in:
- Simulations of non-stationary processes;
- Solving equation systems in comprehensive simulations of stationary heat cycles.

All thermodynamic properties and derivatives can be calculated from precise fundamental equations.

The aim of the IAPWS Advisory Note No. 3 is to describe how to form and calculate any thermodynamic derivative from IAPWS formulations.

History of the Advisory Note No. 3

- Adoption of the first version in 2007
- Issue of a revision in 2008

Contents of the Advisory Note No. 3

- Description of a formal method for determining any thermodynamic derivative from the formulations:
  - IAPWS-95
  - IAPWS-IF97
  - IAPWS-84 for heavy water
  - IAPWS-06 for ice
  - IAPWS-08 for seawater.
4.3 Determination of Partial Derivatives for IAPWS-IF97 Regions 2, 2 meta, and 5

The formulae for calculating the properties \( v, s, c_p, \alpha_v, \) and \( \kappa_T \) of Table 2 from the dimensionless Gibbs free energy equations \( \gamma(\pi, \tau) = \gamma_o(\pi, \tau) + \gamma'(\pi, \tau) \) and its derivatives of regions 2, 2 meta, and 5 of the "IAPWS Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam" (IAPWS-IF97, Revision 2007) [3] are

\[
\begin{align*}
v &= \frac{RT}{p} \left( \gamma_o^0 + \gamma_o^t \right), \\
s &= \frac{R}{T} \left[ \tau \left( \gamma_o^0 + \gamma_o^t \right) - \left( \gamma^0 + \gamma^t \right) \right], \\
c_p &= -RT^2 \left( \gamma_o^0 + \gamma_o^t \right), \\
\alpha_v &= \frac{1}{T} \left( \frac{1 + \pi \gamma_o^t - \tau \pi \gamma_o^t}{1 + \pi \gamma_o^t} \right), \\
\kappa_T &= \frac{1}{p} \left( 1 - \pi^2 \gamma_o^t \right), \\
\end{align*}
\]

(7)

where \( \gamma = g(RT), \pi = p / p^*, \) and \( \tau = T^* / T \) with the specific gas constant \( R \) and the reducing parameters \( p^*, T^* \). The equations \( \gamma_o(\pi, \tau), \gamma'(\pi, \tau) \) and their derivatives which were abbreviated in Eq. (7) as follows:

\[
\begin{align*}
\gamma_o^\pi &= \left( \frac{\partial \gamma_o}{\partial \pi} \right)_T, \\
\gamma_o^\tau &= \left( \frac{\partial \gamma_o}{\partial \tau} \right)_\pi, \\
\gamma_o^\pi^\pi &= \left( \frac{\partial^2 \gamma_o}{\partial \pi^2} \right)_T, \\
\gamma_o^\tau^\tau &= \left( \frac{\partial^2 \gamma_o}{\partial \tau^2} \right)_\pi, \\
\gamma_o^\pi^\tau &= \left( \frac{\partial^2 \gamma_o}{\partial \pi \partial \tau} \right)_T, \\
\gamma_o^\tau^\pi &= \left( \frac{\partial^2 \gamma_o}{\partial \tau \partial \pi} \right)_\tau,
\end{align*}
\]

where the value for \( R \) and the values for \( p^*, T^* \) for each of the regions 2, 2 meta, and 5 are given in [3].